

## 1. Details of Module and its structure

| Module Detail     |  |
|-------------------|--|
| Subject Name      | Physics  |
| Course Name       | Physics 04 (Physics Part-2, Class XII)   |
| Module Name/Title | Unit-08, Module-03: Understanding Bohr's Model for hydrogen atom<br>Chapter-12: Atoms  |
| Module Id         | leph_201203_eContent   |
| Pre-requisites    | Atomic structure, electrons, protons, neutrons, nucleus, Rutherford experiment as done up to secondary level, limitations of Rutherford's atomic model, line spectrum of hydrogen, Bohr's postulates, radius of stationary orbits, energy associated with each stationary state  |
| Objectives        | <p>After completion of this module the learner will be able to :</p> <ul style="list-style-type: none"> <li>• Know about the origin of the line Spectrum of hydrogen atom</li> <li>• Appreciate de Broglie's explanation of Bohr's second postulate of quantisation</li> <li>• Understand Limitations of Bohr model</li> <li>• Explain formation of energy bands on the basis of Pauli's Exclusion Principle and Heisenberg's uncertainty principle</li> </ul> |
| Keywords          | <p>Electron orbit, radius of orbits, velocity of electron in an orbit</p> <p>Bohr's postulates, -13.6 eV energy, eV, limitations of Bohr model energy level questions</p>  |

## 2. Development Team

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**1. UNIT SYLLABUS**

**Unit 8 Atoms and Nuclei**

**Chapter 12 Atoms**

Alpha particle scattering experiment, Rutherford's model of atom, Bohr model, energy levels, hydrogen spectrum

**Chapter 13 Nuclei**

Composition and size of nucleus, radioactivity, alpha, beta and gamma particles/rays and their properties, radioactive decay laws

Mass energy relations, mass defect, binding energy per nucleon and its variation with mass number, nuclear fission and nuclear fusion

**2. MODULE WISE DISTRIBUTION OF UNIT SYLLABUS**

**7 MODULES**

|                 |  |
|-----------------|--|
| <b>Module 1</b> | <ul style="list-style-type: none"> <li>• Introduction</li> <li>• Early models of atom</li> <li>• Alpha particle scattering and Rutherford's Nuclear model of atom</li> <li>• Alpha particle trajectory</li> <li>• Results and interpretations</li> </ul> |
|-----------------|--|

|                 |   |
|-----------------|---|
|                 | <ul style="list-style-type: none"> <li>• Size of nucleus</li> <li>• What Rutherford's model could not explain</li> </ul>  |
| <b>Module 2</b> | <ul style="list-style-type: none"> <li>• Bohr's model of hydrogen atom</li> <li>• Bohr's postulates</li> <li>• Electron orbits, what do they look like?</li> <li>• Radius of Bohr orbits</li> <li>• Energy levels, Energy states, energy unit eV</li> <li>• Lowest energy -13.6 eV interpretation</li> <li>• Velocity of electrons in orbits</li> </ul> |
| <b>Module 3</b> | <ul style="list-style-type: none"> <li>• The line Spectrum of hydrogen atom</li> <li>• de Broglie's explanation of Bohr 's second postulate of quantisation</li> <li>• Departures from Bohr model energy bands</li> <li>• Pauli's Exclusion Principle and Heisenberg's uncertainty principle leading to energy bands</li> </ul>                         |
| <b>Module 4</b> | <ul style="list-style-type: none"> <li>• Atomic masses and composition of nucleus</li> <li>• discovery of neutron</li> <li>• size of nucleus</li> <li>• nuclear forces</li> <li>• energy levels inside the nucleus</li> </ul>   |
| <b>Module 5</b> | <ul style="list-style-type: none"> <li>• Mass and energy, Einstein's relation <math>E = mc^2</math></li> <li>• Mass defect</li> <li>• MeV</li> <li>• Nuclear binding energy</li> <li>• Binding energy per nucleon as a function of mass number</li> <li>• Understanding the graph and interpretations from it</li> </ul>                                |
| <b>Module 6</b> | <ul style="list-style-type: none"> <li>• Radioactivity</li> <li>• Laws of radioactivity</li> <li>• Half life</li> <li>• Rate of decay -disintegration constant</li> <li>• Alpha decay</li> <li>• Beta decay</li> <li>• Gamma decay</li> </ul>   |
| <b>Module 7</b> | <ul style="list-style-type: none"> <li>• Nuclear energy</li> <li>• Fission</li> <li>• Controlled fission reaction</li> <li>• Nuclear Reactor</li> <li>• India atomic energy programme</li> <li>• Nuclear Fusion – energy generation in stars</li> <li>• controlled thermonuclear fusion</li> </ul>  |

## MODULE – 3

## 3. WORDS YOU MUST KNOW

**Atoms:** Atoms are the fundamental building blocks of matter. The existence of different kinds of matter is due to different atoms constituting them.

**Molecules:** A molecule can be defined as the smallest particle of an element or a compound that is capable of an independent existence and shows all the properties of that substance. Atoms of the same element or of different elements can join together to form. The molecules of an element are constituted by the same type of atoms

**Charge:** One of the first indications that atoms are not indivisible comes from studying static electricity

**Electron:** The electron is a subatomic particle, symbol  $e^-$  or  $\beta^-$ , whose electric charge is negative one

**Proton:** A *proton* is a subatomic particle, symbol  $p$  or  $p^+$ , with a positive electric charge of  $+1e$  and a mass slightly less than that of a neutron

**Neutron:** A subatomic particle of about the same mass as a proton but without an electric charge, present in all atomic nuclei except those of ordinary hydrogen.

**Atomic mass:**

One atomic mass unit is a mass unit equal to exactly one-twelfth (1/12th) the mass of one atom of carbon-12. The relative atomic masses of all elements have been found with respect to an atom of carbon-12

**Molecular mass** of a substance is the sum of the atomic masses of all the atoms in a molecule of the substance. It is therefore the relative mass of a molecule expressed in atomic mass units (u)

**The mass of 1 mole of a substance** is equal to its relative atomic or molecular mass in grams. The atomic mass of an element gives us the mass of one atom of that element in atomic mass units (u) **Molar mass** Mass of 1 mole of a substance is called its molar mass.

**Avogadro constant**  $6.022 \times 10^{23}$  is defined as the number of atoms in exactly 12 g of carbon-12.

**The mole** is the amount of substance that contains the same number of particles (atoms/ ions/ molecules/ formula units etc.) as there are atoms in exactly 12 g of carbon-12.

**Rutherford's model of an atom:** Ernest Rutherford was interested in knowing how the electrons are arranged within an atom. Rutherford designed an experiment for this. In this experiment, fast moving alpha( $\alpha$ )-particles were made to fall on a thin gold foil.

On the basis of alpha particle scattering Rutherford proposed the following model of an atom

- i) **An atom consists of a small and massive central core in which the entire positive charge and almost entire mass of the atom are concentrated. The core is called the nucleus**
- ii) **The size of the nucleus is very small  $\sim 10^{-15}$  m as compared to the size of the atom  $\sim 10^{-10}$  m**
- iii) **The nucleus is surrounded by suitable number of electrons so that the atom remains neutral**
- iv) **The electrons revolve around the nucleus in orbits as the planets around the sun the centripetal force is provided by the electrostatic attraction between the electrons and the nucleus**

**Limitations of Rutherford model of the atom:** According to electromagnetic theory an accelerated charged particle must radiate electromagnetic energy. An electron revolving around the nucleus is under continuous acceleration towards the centre as they are revolving in a circle.

It should thus continuously lose energy and move in orbits with gradually decreasing radii and finally collapse into the nucleus. But the nucleus is stable.

**eV:** kinetic energy gained by an electron when subjected to a potential difference of 1 volt

**MeV:**  $10^6$  eV

**Orbit:** supposed track of electron inside the atom

**Bohr's postulates:**

- (i) **Bohr's first postulate an electron in an atom could revolve in certain stable orbits without the emission of radiant energy, these are called the stationary states of the atom.**
- (ii) **Bohr's second postulate defines these stable orbits.**

This postulate states that **the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of  $h/2\pi$**

where  $h$  is the Planck's constant ( $= 6.6 \times 10^{-34}$  J s).

Thus the angular momentum ( $L$ ) of the orbiting electron is quantised.

That is  $L = nh / 2\pi$

- (iii) **It states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy**

**Spectrum:** Spread of colours when a polychromatic light is dispersed. by refraction a prism, interference, or **diffraction**

**Emission line spectrum:** Lines corresponding to discrete wavelengths obtained in spectrum of light emitted by a source

**Absorption line spectrum:** lines corresponding to discrete wavelengths absorbed in spectrum of white light, when passed through a gas

**Rydberg constant is  $1.097 \times 10^7 \text{ m}^{-1}$ .**

**Balmer formula:**  $\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$  where  $\lambda$  is the **wavelength**  $R$  is a constant called the **Rydberg constant**, and  $n$  may have integral values 3, 4, 5, etc.

**Lyman, Paschen, Brackett, and Pfund series**

#### 4. INTRODUCTION

We have learnt about the Bohr Model of atom

This model has its place in the **history of quantum mechanics** and particularly in explaining the structure of an atom.

It has become a milestone since Bohr introduced the revolutionary idea of **definite energy orbits for the electrons**, contrary to the classical picture requiring an accelerating particle to radiate.

Bohr also introduced the idea of **quantisation of angular momentum of electrons** moving in definite orbits. Thus it was a semi-classical picture of the structure of atom.

Now with the development of **quantum mechanics**, we have a better understanding of the structure of atom. Solutions of the **Schrödinger wave equation** assign a wave-like description to the electrons bound in an atom due to attractive forces of the protons.

**An orbit of the electron in the Bohr model is the circular path of motion of an electron around the nucleus.** But according to quantum mechanics, we cannot associate a definite path with the motion of the electrons in an atom. We can only talk about the **probability of finding an electron in a certain region of space around the nucleus.**

This probability can be inferred from the one-electron wave function called the **orbital**. This function depends only on the coordinates of the electron.

It is therefore essential that we understand the subtle differences that exist in the two models:

Bohr model is valid for only **one-electron atoms/ions**; an energy value, assigned to each orbit, depends on the principal quantum number  $n$  in this model. We know that energy associated with a stationary state of an electron depends on  $n$  only, for one-electron atoms/ions.

For a **multi-electron atom**, this is not true.

The solution of **the Schrödinger wave equation**, obtained for hydrogen-like atoms/ions, called the wave function, gives information about the probability of finding an electron in various regions around the nucleus. This *orbital* has no resemblance whatsoever with the *orbit* defined for an electron in the Bohr model. (This is beyond our scope of present study)

**In this module, we will study the energy states occupied by electrons, inside the atom.**

**ENERGY STATES/LEVELS**

**In module 2 of this unit we calculated the energy values; radii of probable stationary orbits for a hydrogen atom. You will recall some of these considerations**

- The energy of an atom is the least (largest negative value) when its electron is revolving in an orbit closest to the nucleus i.e., the one for which  $n = 1$ . For  $n = 2, 3, \dots$  the absolute value of the energy  $E$  is smaller, hence the energy is progressively larger in the outer orbits.
- The lowest state of the atom, called the **ground state**, is that of the lowest energy, with the electron revolving in the orbit of smallest radius, the **Bohr radius,  $a_0$** .

The energy of this state ( $n = 1$ ),  $E_1$  is  $-13.6$  eV.

- Therefore, the minimum energy required to free the electron from the ground state of the hydrogen atom is  $13.6$  eV.

It is called the **ionisation energy** of the hydrogen atom.

**This prediction of the Bohr's model is in excellent agreement with the experimental value of ionisation energy.**

At room temperature, most of the hydrogen atoms are in **ground state**.

When a hydrogen atom receives energy by processes such as electron collisions, the atom may acquire sufficient energy to raise the electron to higher energy states. The atom is then said to be in an **excited state**.

From

$$E_n = -\frac{13.6}{n^2} J$$

$n$  is called the **principle quantum number**.

for  $n = 2$ ; the energy  $E_2$  is  $-3.40$  eV.

It means that the energy required to excite an electron in hydrogen atom to its **first excited state**, is an energy equal to

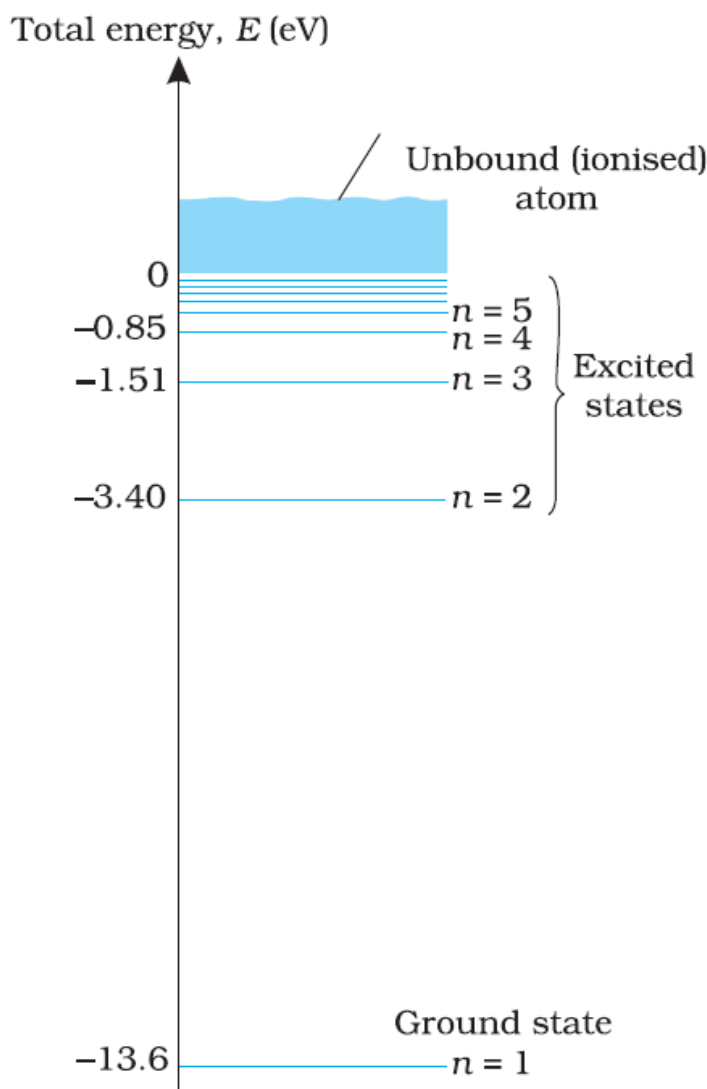
$$E_2 - E_1 = -3.40 \text{ eV} - (-13.6) \text{ eV} = 10.2 \text{ eV}.$$

Similarly,  $E_3 = -1.51$  eV and  $E_3 - E_1 = 12.09$  eV, or to excite the hydrogen atom from its ground state ( $n = 1$ ) to second excited state ( $n = 3$ ),  $12.09$  eV energy is required, and so on.

**From these excited states the electron can then fall back to a state of lower energy, emitting a photon in the process.**

Thus, as the excitation of hydrogen atom increases (that is as  $n$  increases) the value of minimum energy required to free the electron from the excited atom decreases.

**The energy level diagram\* for the stationary states of a hydrogen atom is given**



*The energy level diagram for the hydrogen atom.*

*The electron in a hydrogen atom at room temperature spends most of its time in the ground state. To ionise a hydrogen atom an electron from the ground state, 13.6 eV of energy must be supplied. (The horizontal lines specify the presence of allowed energy states.)*

The principal quantum number  $n$  labels the stationary states in the ascending order of energy.

In this diagram, **the highest energy state corresponds to  $n = \infty$  and has an energy of 0 eV.**

This is the energy of the atom when the electron is completely removed ( $r = \infty$ ) from the nucleus and is at rest.

**Observe how the energies of the excited states come closer and closer together as  $n$  increases.**

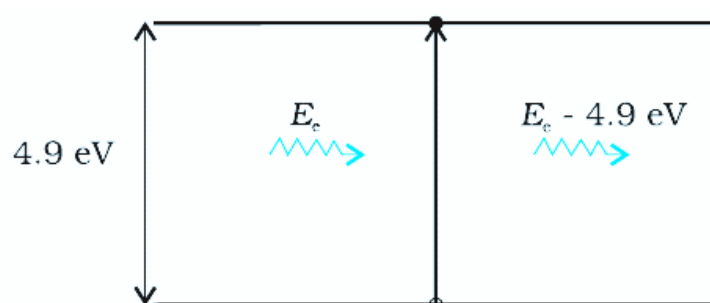


## 5. FRANCK – HERTZ EXPERIMENT

### Why do atoms emit radiation?

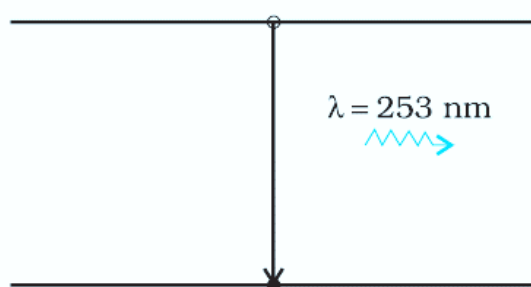
The existence of discrete energy levels in an atom was directly verified in 1914 by James Franck and Gustav Hertz. They studied the spectrum of mercury vapour when electrons having different kinetic energies passed through the vapour. The electron energy was varied by subjecting the electrons to electric fields of varying strength. The electrons collide with the mercury atoms and can transfer energy to the mercury atoms. This can only happen when the energy of the electron is higher than the energy difference between an energy level of Hg occupied by an electron and a higher unoccupied level (see Figure).

For instance, the difference between an occupied energy level of Hg and a higher unoccupied level is 4.9 eV.



**If an electron of having energy of 4.9 eV or more passes through mercury, an electron in mercury atom can absorb energy from the bombarding electron and get excited to the higher level. The colliding electron's kinetic energy would reduce by this amount.**

The excited electron would subsequently fall back to the ground state by emission of radiation



The wavelength of emitted radiation is:

$$\lambda = \frac{hc}{E} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{4.9 \times 1.6 \times 10^{-19}} = 253 \text{ nm}$$

By direct measurement, Franck and Hertz found that the emission spectrum of mercury has a line corresponding to this wavelength.

For this experimental verification of Bohr's basic ideas of discrete energy levels in atoms and the process of photon emission, Frank and Hertz were awarded the Nobel Prize in 1925.

## 6. THE LINE SPECTRA OF THE HYDROGEN ATOM

According to the third postulate of Bohr's model, when an atom makes a transition from the higher energy state with quantum number  $n_i$  to the lower energy state with quantum number  $n_f$  ( $n_f > n_i$ ),

the difference of energy is carried away by a photon of frequency  $\gamma_{i-f}$  such that

$$h \gamma_{i-f} = E n_i - E n_f$$

$$\gamma_{i-f} = \frac{m e^4}{8 \epsilon_0^2 h^2} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

This Equation is the Rydberg formula, for the spectrum of the hydrogen atom. In this relation, if we take  $n_f = 2$  and  $n_i = 3, 4, 5, \dots$ ,

it reduces to a form similar to the expression for the Balmer series.

$$\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$$

The **Rydberg constant  $R$**  is readily identified to be

$$R = \frac{m e^4}{8 \epsilon_0^2 h^3 c}$$

If we insert the values of various constants, we get  $R = 1.03 \times 10^7 \text{ m}^{-1}$

This is a value very close to the value ( $1.097 \times 10^7 \text{ m}^{-1}$ ) obtained from the empirical Balmer formula.

**This agreement between the theoretical and experimental values of the Rydberg constant provided a direct and striking confirmation of the Bohr's model.**

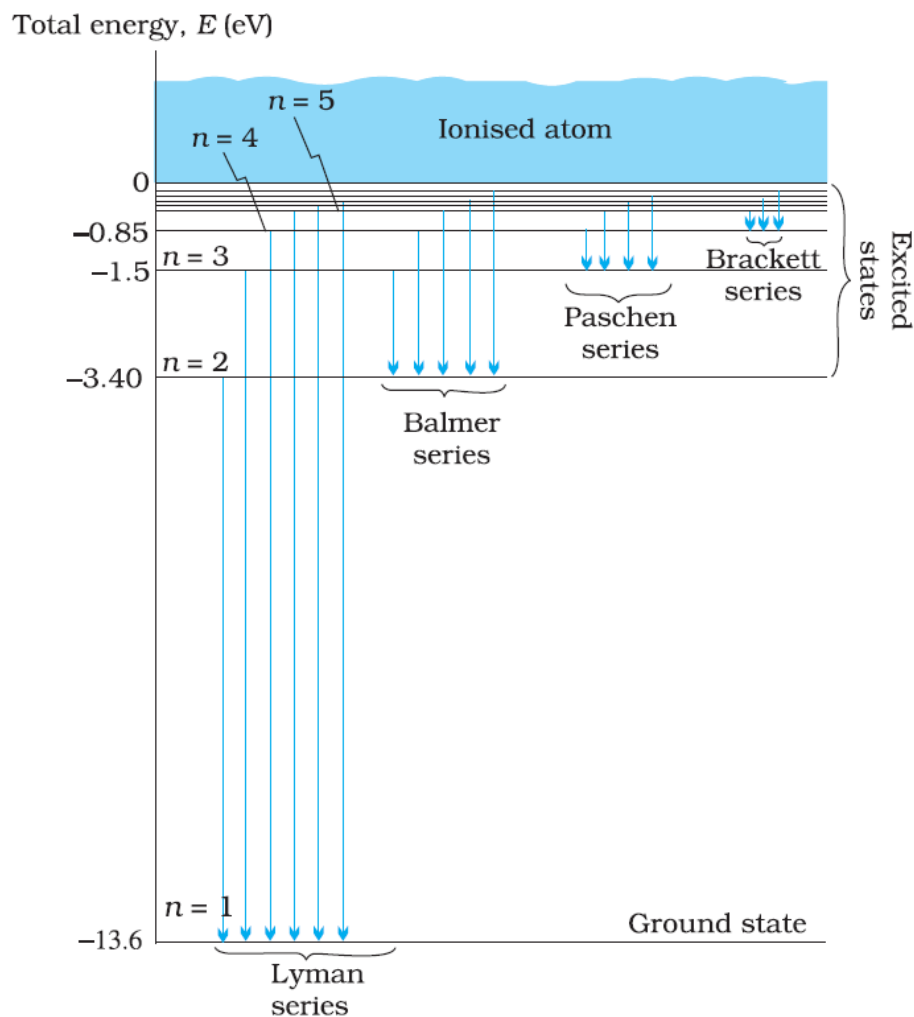
Since both  $n_f$  and  $n_i$  are integers, this immediately shows that in transitions between different atomic levels, light is radiated in various discrete frequencies.

For hydrogen spectrum, the Balmer formula corresponds to  $n_f = 2$  and  $n_i = 3, 4, 5, \text{ etc.}$

The results of the Bohr's model suggested the presence of other series spectra for hydrogen atom—those corresponding to transitions resulting from  $n_f = 1$  and  $n_i = 2, 3, \text{ etc.}$ ;  $n_f = 3$  and  $n_i = 4, 5, \text{ etc.}$ , and so on.

Such series were identified in the course of spectroscopic investigations and are known as the Lyman, Balmer, Paschen, Brackett, and Pfund series.

**Study the electronic transitions corresponding to these series from the Figure shown**



Line spectra originate in transitions between energy levels.

The various lines in the atomic spectra are produced when electrons jump from higher energy state to a lower energy state and photons are **emitted**. These spectral lines are called emission lines. But when an atom absorbs a photon that has precisely the same energy needed by the electron in a lower energy state to make transitions to a higher energy state, the process is called **absorption**.

Thus if photons with a continuous range of frequencies pass through a rarefied gas and then are analysed with a spectrometer, a series of dark spectral absorption lines appear in the continuous spectrum. The dark lines indicate the frequencies that have been absorbed by the atoms of the gas.

The explanation of the hydrogen atom spectrum provided by Bohr's model was a brilliant achievement, which greatly stimulated progress towards the modern quantum theory. In 1922, Bohr was awarded Nobel Prize in Physics.

### EXAMPLE

Using the Rydberg formula, calculate the wavelengths of the first four spectral lines in the Lyman series of the hydrogen spectrum.

**SOLUTION**

The Rydberg formula is

$$\gamma_{if} = \frac{me^4}{8\epsilon_0^2 h^2} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

The wavelengths of the first four lines in the Lyman series correspond to transitions from  $n_i = 2, 3, 4, 5$  to  $n_f = 1$ . We know that

$$\frac{me^4}{8\epsilon_0^2 h^2} = 13.6 \text{ eV} = 21.78 \times 10^{-19} \text{ J}$$

$$\begin{aligned} \lambda_{i1} &= \frac{hc}{21.78 \times 10^{-19} \left( \frac{1}{1} - \frac{1}{n_i^2} \right)} m \\ &= \frac{6.625 \times 10^{-34} \times 3 \times 10^8 \times n_i^2}{21.78 \times 10^{-19} \times (n_i^2 - 1)} = \frac{0.9134 n_i^2 \times 10^{-7}}{(n_i^2 - 1)} m \end{aligned}$$

$$= 913.4 \frac{n_i^2}{(n_i^2 - 1)} \text{ \AA}$$

Substituting  $n_i = 2, 3, 4, 5$ ,

$$\text{we get } \lambda_{21} = 1218 \text{ \AA}$$

$$\lambda_{31} = 1028 \text{ \AA}$$

$$\lambda_{41} = 974.3 \text{ \AA}$$

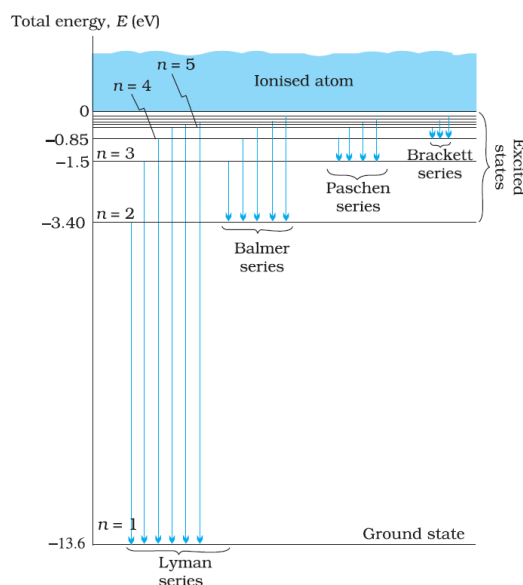
$$\lambda_{51} = 951.4 \text{ \AA}$$

**EXAMPLE**

A 12.1 eV beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

**SOLUTION**

Energy absorbed will take the electron in the ground state to  $n = 3$



$$E_3 = 1.5$$

$$E_1 = 13.6$$

$$\Delta E = E_3 - E_1 = 12.1 \text{ eV}$$

The transitions possible from n- 3 are

$n_3$  to  $n_2$

$n_2$  to  $n_1$

$n_3$  to  $n_1$

$$\text{wavelength emitted is given by } \lambda = \frac{hc}{\Delta E} = \frac{6.67 \times 10^{-34} \times 3 \times 10^8}{\Delta E} = \frac{12.43 \times 10^{-7}}{\Delta E}$$

$$\text{for } n_3 \text{ to } n_2 \quad \Delta E = -1.5 - (-3.40) = 1.9 \text{ eV}$$

$$\lambda = \frac{12.43 \times 10^{-7}}{\Delta E} = \frac{12.43 \times 10^{-7}}{1.9 \text{ eV}} = 6.542 \times 10^{-7} \text{ m}$$

### TRY THESE

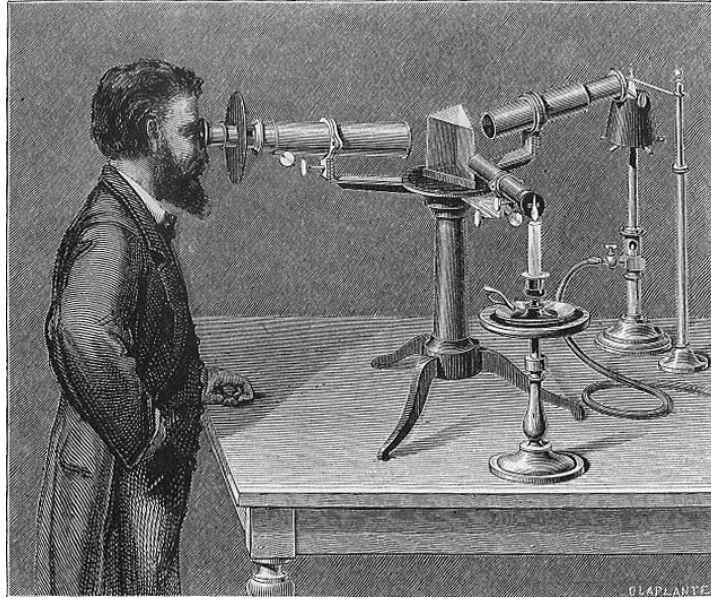
- What is the energy level  $E_6$  for an electron in orbit  $n = 6$ , and how much energy is given off if this electron jumps to orbit  $n = 2$ ?
- What kind of light does a hydrogen atom give off when an electron jumps from an orbit with  $n > 1$  down to orbit  $n = 1$ ? Is this light visible?

### 7. HOW DO WE OBSERVE THE SPECTRUM?

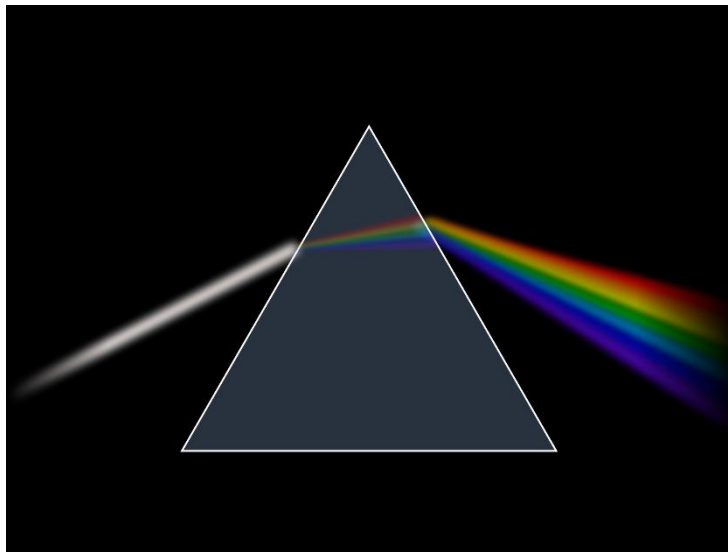
We have learnt of three ways to obtain a spectrum

- Refraction of polychromatic light through a triangular equilateral prism,
- Young's double slit interference pattern using polychromatic light or
- By diffraction.

Spectroscope and spectrometer are used to view the spectrum and make measurements

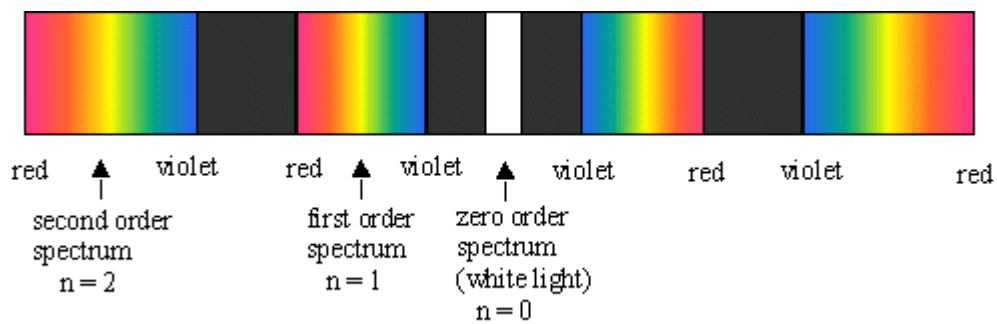
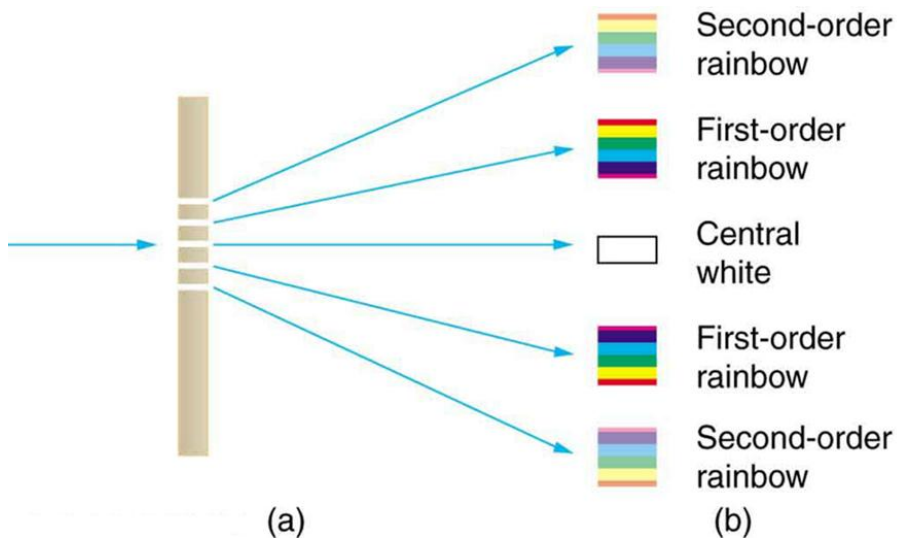
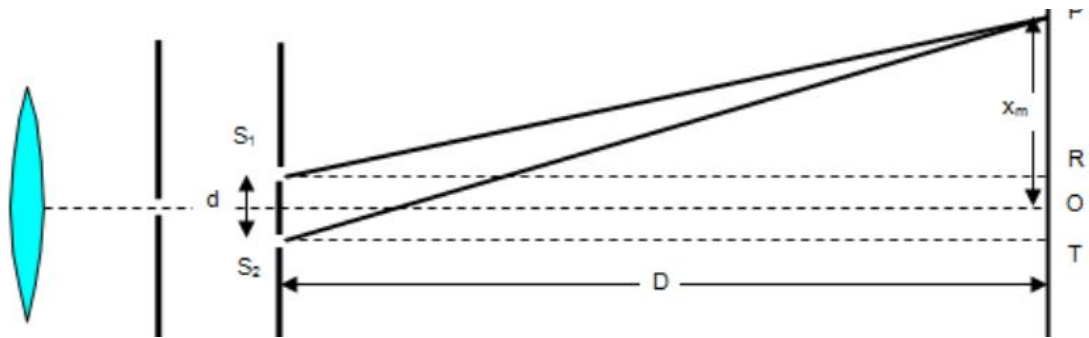


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<https://upload.wikimedia.org/wikipedia/commons/thumb/7/76/Prism-rainbow-black-2.svg/2000px-Prism-rainbow-black-2.svg.png>

**Young's double slit arrangement and fringes produced by polychromatic light**



[http://philschatz.com/physics-book/resources/Figure\\_28\\_04\\_01a.jpg](http://philschatz.com/physics-book/resources/Figure_28_04_01a.jpg)

### WHY IS THE SPECTRUM CALLED -A LINE SPECTRUM?

The spread of colours is visible clearly only when there is high resolution. Instead of point opening for the polychromatic light we usually have a vertical slit  
The spectrum then is a collection of vertical bright lines signature of the source of light.

### 8. DE BROGLIE'S EXPLANATION OF BOHR'S SECOND POSTULATE OF QUANTISATION

Of all the postulates, Bohr made in his model of the atom, perhaps the most puzzling is his second postulate

**It states that the angular momentum of the electron orbiting around the nucleus is quantised**

**that is,  $L_n = n \frac{h}{2\pi}$  ; (  $n = 1, 2, 3 \dots$ ).**

### WHY SHOULD THE ANGULAR MOMENTUM HAVE ONLY THOSE VALUES THAT ARE INTEGRAL MULTIPLES OF $h/2\pi$ ?

The French physicist Louis de Broglie explained this puzzle in 1923, ten years after Bohr proposed his model.

We studied about the de Broglie's hypothesis that material particles, such as electrons, also have a wave nature. C. J. Davisson and L. H. Germer later experimentally verified the wave nature of electrons in 1927.

Louis de Broglie argued that the electron in its circular orbit, as proposed by Bohr, must be seen as a particle wave. In analogy to waves travelling on a string, particle waves too can lead to **standing waves under resonant conditions.**

we know that, when a string is plucked, a vast number of wavelengths are excited. However only those wavelengths survive which have nodes at the ends and form the standing wave in the string. It means that in a string, standing waves are formed when the total distance travelled by a wave down the string and back is one wavelength, two wavelengths, or any integral number of wavelengths.

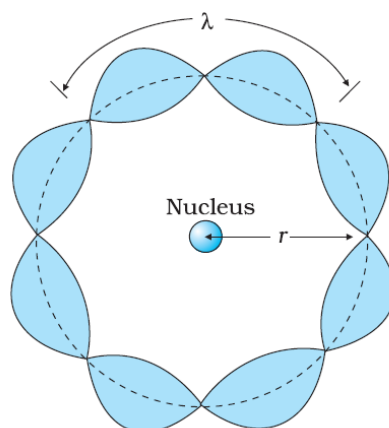
Waves with other wavelengths interfere with themselves upon reflection and their amplitudes quickly drop to zero. For an electron moving in  $n$ th circular orbit of radius  $r_n$

the total distance is the circumference of the orbit,  $2\pi r_n$ .

Thus

$$2\pi r_n = n\lambda, n = 1, 2, 3\dots$$





**A standing wave is shown on a circular orbit where four de Broglie wavelengths fit into the circumference of the orbit.**

It illustrates a standing particle wave on a circular orbit for  $n = 4$ , i.e.,

$$2\pi r_n = 4\lambda,$$

where  $\lambda$  is the de Broglie wavelength of the electron moving in  $n$ th orbit.

we have  $\lambda = h/p$ , where  $p$  is the magnitude of the electron's momentum.

If the speed of the electron is much less than the speed of light, the momentum is  $mv_n$ .

Thus,  $\lambda = h/mv_n$ .

we have

$$2\pi r_n = n h/mv_n$$

or

$$m v_n r_n = nh/2\pi$$

**This is the quantum condition proposed by Bohr for the angular momentum of the electron**

we have seen that this equation is the basis of explaining the discrete orbits and energy levels in hydrogen atom.

Thus de Broglie hypothesis provided an explanation for Bohr's second postulate for the quantisation of angular momentum of the orbiting electron.

The quantised electron orbits and energy states are due to the wave nature of the electron and only resonant standing waves can persist.

## 9. LIMITATIONS OF BOHR'S MODEL

Bohr's model, involving classical trajectory picture (planet-like electron orbiting the nucleus), correctly predicts the gross features of the hydrogenic atoms the frequencies of the radiation emitted or selectively absorbed. **This model however has many limitations.**

**Some are:**

(i) **The Bohr model is applicable to hydrogenic atoms. It cannot be extended even to mere two electron atoms such as helium. The analysis of atoms with more than one electron was attempted on the lines of Bohr's model explained the hydrogen atom, but did not meet with any success with others.**

**Difficulty lies in the fact that each electron interacts not only with the positively charged nucleus but also with all other electrons.**

**The formulation of Bohr model involves electrical force between positively charged nucleus and electron. It does not include the electrical forces between electrons which necessarily appear in multi-electron atoms.**

(ii) **While the Bohr's model correctly predicts the frequencies of the light emitted by hydrogen atoms, the model is unable to explain the relative intensities of the frequencies in the spectrum.**

**In emission spectrum of hydrogen, some of the visible frequencies have weak intensity, while some others are strong. Why? Experimental observations depict that some transitions are more favoured than others.**

**Bohr's model is unable to account for the intensity variations.**

**Bohr's model presents an elegant picture of an atom and cannot be generalised to complex atoms. For complex atoms we have to use a new and radical theory based on Quantum Mechanics, which provides a more complete picture of the atomic structure.**

### **HEISENBERG'S UNCERTAINTY PRINCIPLE**

According to the principle 'It is impossible to measure simultaneously the position and momentum of a small particle with absolute accuracy or certainty'.

The product of the uncertainty in the position (  $\Delta x$  ) and the uncertainty in the momentum ( $\Delta p$ ) is always constant and is equal to or greater than  $h/4\pi$ , where  $h$  is the Planck's Constant  
ie.

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

If uncertainty in position is  $10^{-4}$ , uncertainty in velocity will be 0.1 m/s.

**Bohr's concept of fixed circular path with definite position and momentum of electron have been replaced by stating that the electron has the probability of having a given position and momentum.**

### **PAULI EXCLUSION PRINCIPLE**

States that, in an atom or molecule, no two electrons can have the same four electronic quantum numbers ( refer to your chemistry syllabus). As an orbital can contain a maximum of only two electrons, the two electrons must have opposing spins.

**Pauli Exclusion Principle**, asserts that no two electrons in an atom can be at the same time in the same state or configuration.

This was proposed in (1925) by the Austrian physicist Wolfgang Pauli to account for the observed patterns of light emission from atoms.

**In bulk matter the atoms and molecules are bound to each other. The electron orbits of close atoms modify, making each electron in a state to have slightly different energy and form a band of allowed energy states.**

**Pure line emission spectrum is obtained from gases at incandescent temperature at low pressure. Higher pressure line spectra tend towards band spectra, or we can say the unique wavelengths in the spectral lines are a band of nearly equal wavelengths.**

## 10. SUMMARY

- To explain the line spectra emitted by atoms, as well as the stability of atoms, Neil's Bohr proposed a model for hydrogenic (single electron) atoms. He introduced three postulates and laid the foundations of quantum mechanics:
  - (a) In a hydrogen atom, an electron revolves in certain stable orbits (called stationary orbits) without the emission of radiant energy.
  - (b) The stationary orbits are those for which the angular momentum is some integral multiple of  $h/2\pi$ . (Bohr's quantisation condition.) That is  $L = nh/2\pi$ , where  $n$  is an integer called a quantum number.
  - (c) The third postulate states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states.
- The frequency ( $\gamma$ ) of the emitted photon is then given by  $h\gamma = E_i - E_f$
- An atom absorbs radiation of the same frequency the atom emits, in which case the electron is transferred to an orbit with a higher value of  $n$ .

$$E_i + h\gamma = E_f$$

As a result of the quantisation condition of angular momentum, the electron orbits the nucleus at only specific radii. For a hydrogen atom it is given by

$$r_n = \left(\frac{n^2}{m}\right) \left(\frac{h}{2\pi}\right)^2 \frac{4\pi\epsilon_0}{e^2}$$

The total energy is

$$\begin{aligned} \text{or } E_n &= -\frac{me^4}{8n^2\epsilon_0^2h^2} \\ &= -13.6 \text{ eV}/n^2 \end{aligned}$$

- The  $n = 1$  state is called ground state. In hydrogen atom the ground state energy is  $-13.6$  eV. Higher values of  $n$  correspond to excited states ( $n > 1$ ). Atoms are excited to these higher states by collisions with other atoms or electrons or by absorption of a photon of right frequency.
- de Broglie's hypothesis that electrons have a wavelength  $\lambda = h/mv$  gave an explanation for Bohr's quantised orbits by bringing in the wave particle duality. The orbits correspond to circular standing waves in which the circumference of the orbit equals a whole number of wavelengths.
- Bohr's model is applicable only to hydrogenic (single electron) atoms. It cannot be extended to even two electron atoms such as helium.
- This model is also unable to explain for the relative intensities of the frequencies emitted even by hydrogenic atoms. Heisenberg's uncertainty principle and Pauli's exclusion principle help energy states to accommodate more electrons and the state then has a band of allowed energies.